FOUR TASKS
Choose one!

$A - 4 = 6 - B$

What can you say about A and B? How do you know?

Draw at least six triangles that have an area of 5 square units.

Draw a _____ whose angles are congruent but whose sides are not congruent.

Create a story and a diagram to illustrate the meaning of $3 \div \frac{1}{2}$.

Choose a task. Decide on an appropriate grade level. Approach it as a student at that grade level.

- What are the instructional goals of the task?

- How does the task extend math standards?

- Is the task deep? Is it complex? Why or why not?

- How would you use the task in your classroom?

Extending Math Standards for Gifted Students K–8

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Goals

- Explore examples of deep mathematical tasks.

- Connect the tasks to gifted research and to the Common Core Math Content and Practice Standards.

- Discuss the meaning and significance of deep math tasks and instruction for gifted students.

- Identify principles and techniques for using CCSS-M to enhance learning opportunities for gifted students.

- Apply the techniques to practice generating deep and complex math tasks.

Depth In Math Instruction
for Gifted Students

- *Why* do we need it?

- *How* do we achieve it?

- *What* resources and strategies support it?

Language as a Window into Depth (1)

Standards Retained	Standards Omitted
Divide Calculate (+ − x ÷) Recognize —> write Represented Interpret —> decide (form of remainder) Select (a form of a number) Consider context Solve problem —> story problem Knowledge of place value Use various strategies Relationships Context Reasonableness	Estimate Assess reasonableness

Language as a Window into Depth (2)

Standards Retained	Standards Omitted
Order (verb) Locate on number line	Read / Write (place value) Place value Find (0.1 more / less; 0.01 more / less…) Recognize Generate Equivalent expressions Contexts Round

Language as a Window into Depth (3)
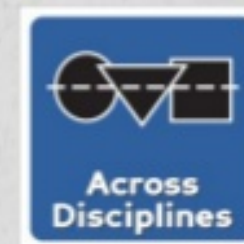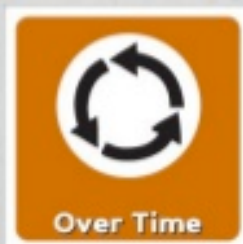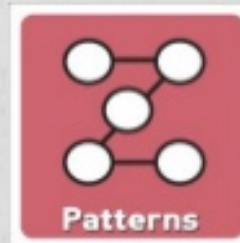
Standards Retained	Standards Omitted
Add/subtract	Model (verb)
Add/subtract/multiply/divide	A variety of representations
Solve problems —> find answers to story problems	Estimate
	Assess reasonableness

Kaplan's Icons

DEPTH & COMPLEXITY ICONS

Trends

Rules

Language of the Discipline

Unanswered Questions

Big Idea

Patterns

Ethics

Over Time

Details

Multiple Perspectives

Across Disciplines
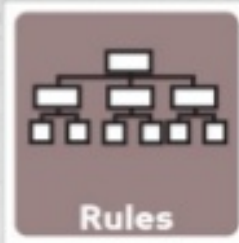
Based upon the work of Sandra Kaplan, USC

NCTM Process Standards

❖ Problem Solving
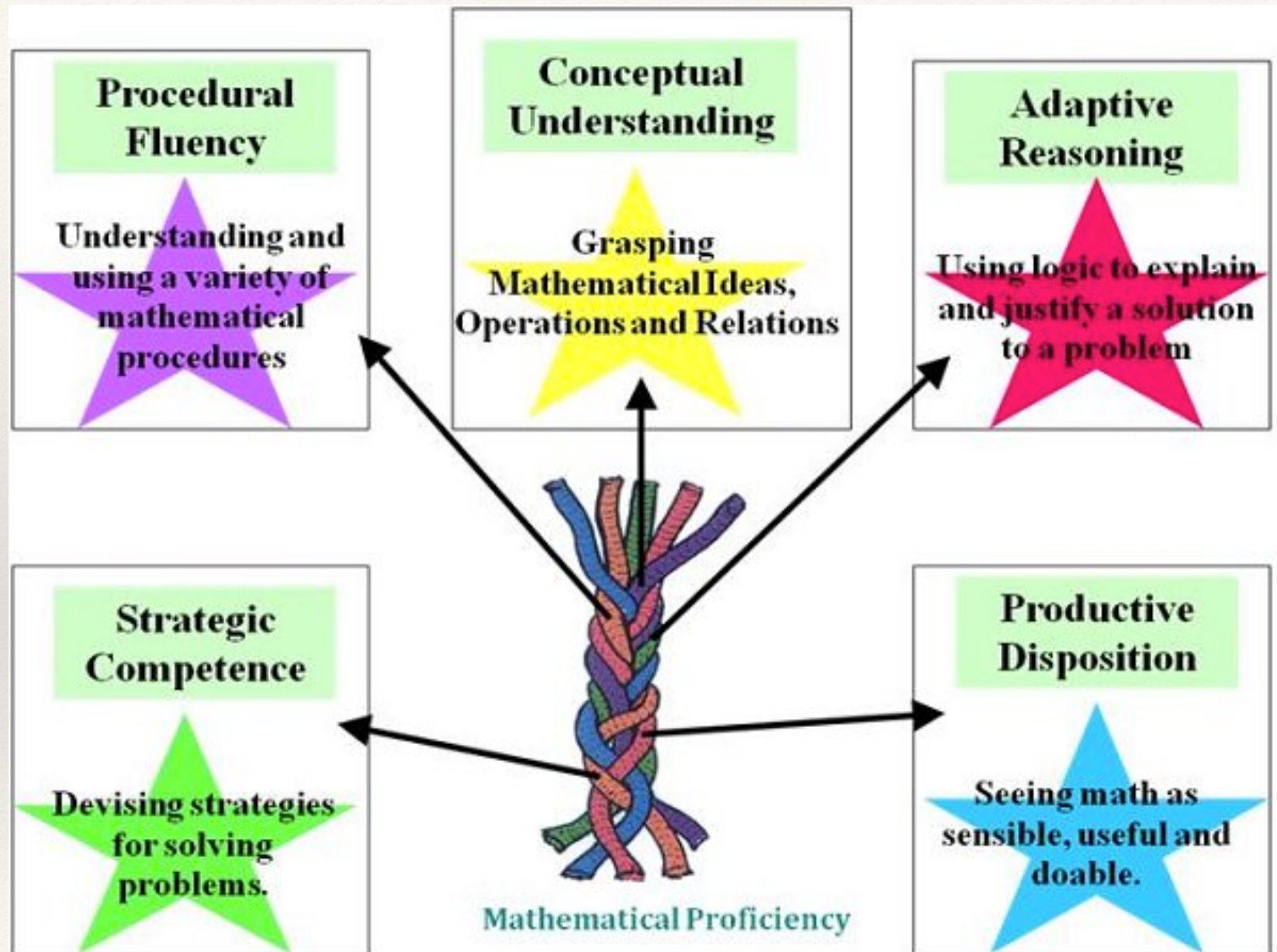
   ❖ Develop, apply, and verify your own strategies to answer questions.

❖ Reasoning and Proof

   ❖ Make and test predictions. Analyze and extend patterns. Justify conclusions.

❖ Communication

   ❖ Organize, record, and present mathematical ideas clearly (orally and in writing).

❖ Connections

   ❖ Recognize relationships among mathematical ideas and between math and other disciplines.

❖ Representations

   ❖ Model math concepts with words, graphs, tables, symbols, pictures, manipulatives, etc.

Math Proficiency Strands

Procedural Fluency
Understanding and using a variety of mathematical procedures

Conceptual Understanding
Grasping Mathematical Ideas, Operations and Relations

Adaptive Reasoning
Using logic to explain and justify a solution to a problem

Strategic Competence
Devising strategies for solving problems.

Productive Disposition
Seeing math as sensible, useful and doable.

Mathematical Proficiency

Kilpatrick, J., Swafford, J., Findell, B. (Ed.). (2001). Adding it up: helping children learn mathematics. Washington, DC: National Academy Press.

CCSS – Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Common Core State Standards for Mathematics, 2010

Connecting Kaplan to Best Practices in Math

Conceptual Understanding	**Big Idea**, Patterns, Trends, Different Perspectives
Procedural Fluency	**Rules**, Ethics, Different Perspectives
Adaptive Reasoning	**Patterns** and **Details**, Trends, Different Perspectives
Strategic Competence	**Unanswered Questions**, Different Perspectives, Ethics
Mathematical Communication	**Language of the Discipline**, Rules, Different Perspectives
Connections	**Across the Disciplines**, Different Perspectives, Patterns
Representations	**Different Perspectives**, Language of the Discipline

Extending the Common Core Math Standards

★ **Increase Depth**

- Focus on the *Practice* standards

- Attend to language in content standards (*understand*, *represent*, *predict*, *interpret*,…)

- Use the *10 Strategies for Increasing Depth*

★ **Increase complexity**

- Use the *Strategy for Increasing Complexity* (more...)

- Provide options for level of complexity

- Integrate with the 10 Strategies for Depth

★ **Increase breadth**
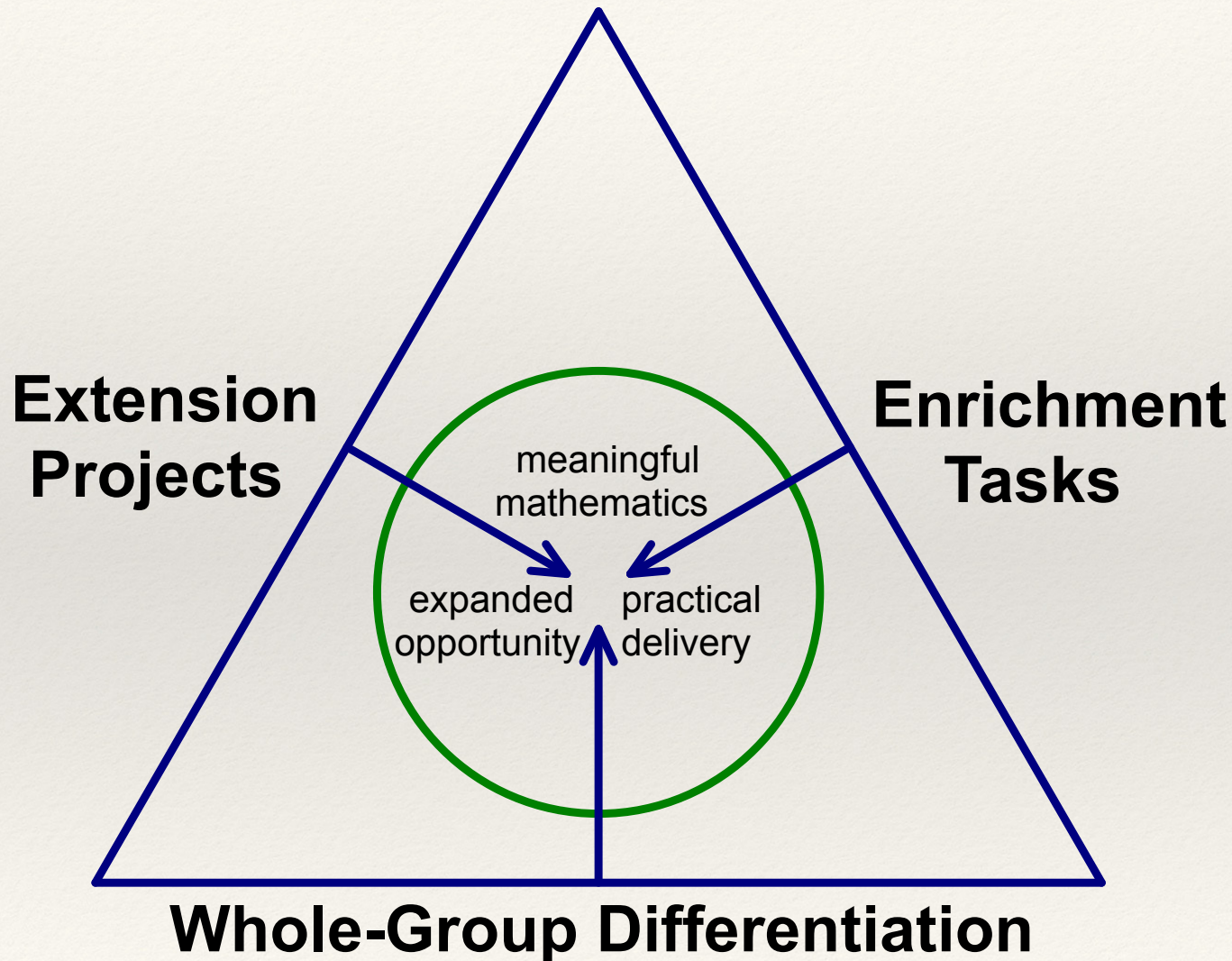
- Explore outside content standards (*practical*, *recreational*, *interdisciplinary*, *advanced*)

Math Instruction for Gifted Students

Extension Projects

Enrichment Tasks

meaningful mathematics

expanded opportunity

practical delivery

Whole-Group Differentiation

10 Strategies
for Increasing Depth of Math Tasks

1. Write a Story.

2. Draw a Picture.

3. Find another way.

4. Explain why.

5. Compare and contrast.

6. Start with the answer.

7. Remove information.

8. Solve to learn.

9. Build a pattern.

10. Ask "What If…?"

1 Strategy
for Increasing the Complexity of Math Tasks

Use *more…*

- digits.

- numbers.

- shapes.

- parts.

- steps.

- definitions.

- categories.

- relationships.

Applying the 10 Strategies for Depth

Original Task: $6 \div \frac{2}{3}$

❖ Strategy 1: Write a story.

❖ Strategy 2: Draw a picture.

❖ Strategy 3: Explain why.

New Task: Write a story problem for $6 \div \frac{2}{3}$. Draw a picture, and

use it show the answer and justify it.

Applying the 10 Strategies for Depth

$$\text{Original Task: } 6 \div \frac{2}{3}$$

❖ **Strategy 4: Find another way.**

  New Task: Find another way to calculate $6 \div \frac{2}{3}$.

Applying the 10 Strategies for Depth

Original Task: $6 \div \frac{2}{3}$

❖ **Strategy 5: Compare and contrast.**

New Task: Create other examples of division equations for which the quotient is greater than the dividend. What do your examples have in common? How are they different?

Applying the 10 Strategies for Depth

Original Task: $6 \div \frac{2}{3}$

❖ **Strategy 6: Start with the answer.**

❖ **Strategy 9: Create a pattern.**

New Task: Write at least four equations in the form

$$whole\ number \div fraction = 9$$

Describe any patterns that you see.

Applying the 10 Strategies for Depth

Original Task: $6 \div \dfrac{2}{3}$

❖ **Strategy 7: Remove information.**

New Task: $6 \div \dfrac{\square}{3}$

Applying the 10 Strategies for Depth

Original Task: $6 \div \frac{2}{3}$

❖ **Strategy 8: Solve to learn.**

❖ **Strategy 3: Explain why.**

New Task: Find the value of $6 \div \frac{2}{3}$. Explain why your answer makes sense.

(Assign the task *before* teaching a procedure for dividing fractions.)

Applying the 10 Strategies for Depth

Original Task: $6 \div \dfrac{2}{3}$

❖ **Strategy 10: Ask "What if…?"**

New Task: What happens to the value of $6 \div \dfrac{2}{3}$ if you

- multiply the dividend by 3?

- multiply the divisor by 3?

- multiply the dividend and the divisor by 3?

- multiply the _____ by _____? (Strategy 7)

Applying the Strategy for Complexity

$$\text{Original Task: } 6 \div \frac{2}{3}$$

Strategy for Complexity: Use more.

❖ Increase the size of the 6, 2, or 3.

❖ Make the numerator a number that is not a factor of 6.

❖ Make the dividend a fraction.

Sample Tasks to Modify

❖ Round (83) to the nearest ten.

❖ Identify the (parallelogram) in the pictures.

❖ Simplify $(\frac{6}{15})$.

❖ Find the area of the (triangle).

❖ Find the prime factorization of (56).

❖ Find the value of $(6 \cdot \frac{2}{3})$.

❖ Find the value: $(\frac{3 \times 10^1}{5 \times 10^{-2}})$.

Resources

- *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

- *Helping Children Learn Mathematics*, by the National Research Council, 2000.

- *Extending the Challenge in Mathematics*, by Linda Sheffield. Corwin Press, 2003.

- *Advanced Common Core Math Explorations* series, by Jerry Burkhart. Prufrock Press, 2014 – 2015

- *Good Questions: Great Ways to Differentiate Math Instruction*, by Marian Small. Teacher's College Press, 2012.

Thank you!

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Advanced Common Core Math Explorations books:

http://www.prufrock.com