Step Up!
Introduction

Topics

- Properties of addition and multiplication
- Strategies for multiplying multi-digit numbers
- Discovering and describing patterns

Materials

- Dot paper or graph paper (to draw pictures of the steps).
- Recommended: Blocks or counters (two different colors are best) to make staircases

What students should know

- Understand that $A \times B$ represents A groups of B or B groups of A.
- Use arrays to represent multiplication of whole numbers.
- Take half of a number (divide it by 2).

How the activity extends math standards

- Use arrays and the meaning of multiplication to develop strategies for multiplying multi-digit numbers.
- Explore properties of addition and multiplication.
- Use arrays to explore complex patterns involving multiplication and addition.

Getting started

Display or hand out copies of the Opener. Start an open-ended discussion by asking students what they *notice* and *wonder*. Use the "I notice/I wonder" T-chart if you like. Give students plenty of time to get comfortable sharing math ideas openly.

The *Noticing and Wondering: Sample Responses* page has ideas to help you guide discussion. Remember that these are only suggestions. The ideas should come mainly from your students. It may be easier for students to notice than to wonder.

If you are not available to lead the opening discussion, ask your students to work in small groups to discuss and write down the things that they notice and wonder. You may check back with them later to help them identify questions to explore.

Once your students understand the problem and have created some questions to explore, they may begin working on answering some of their questions. If you prefer, you may hand out the Directions page so that they have a specific list of questions to answer. However, students need not answer all of these questions, and it is important that they have a chance to answer questions that *they* create as well.

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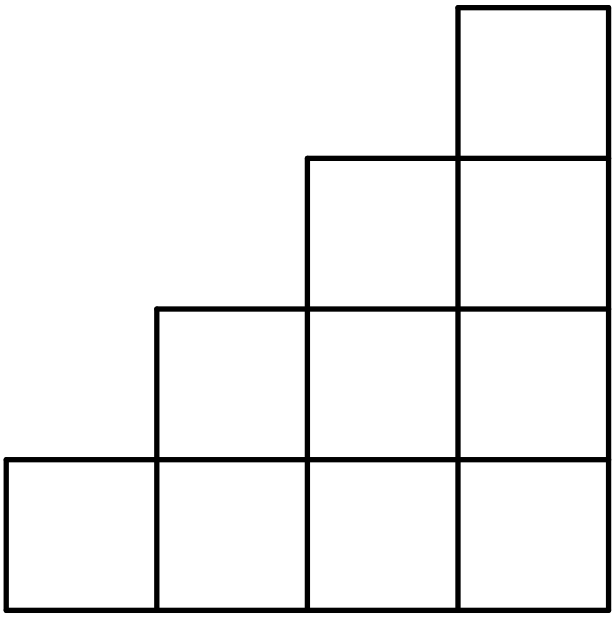
Step Up!
Opener

What do you notice? What do you wonder?



$$1 + 2 + 3 + 4$$

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Name _____

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I notice	I wonder

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Step Up!
Noticing and Wondering: Sample Responses

Note: Students may have many ideas that are not on this page.

1. **Before starting the problem**
   I notice four steps.
   I notice that each step has one more block than the one before.
   I notice that the steps use 10 blocks.
   I notice more than one way to see 1 + 2 + 3 + 4.
   I notice that I can make a rectangle by moving just three blocks.
   I wonder how many blocks it would take to make 10 steps?
   I wonder how many blocks it would take to make 100 or 1000 steps?
   I wonder if I could use steps to find fast a way to add long strings of numbers?
   I wonder how many steps I could make if I had 500 blocks?

2. **While solving the problem**
   I notice that I can make rectangles by moving the blocks from the top half of the steps.
   I notice that making rectangles makes it easier to count the blocks.
   I notice that the problem seems harder when there is an odd number of steps.
   I notice that it can help to add the numbers in a different order.
   I notice that it can help to group the numbers before I add them.
   I notice that if I make a copy of the steps, I can join it to the other steps to make a rectangle.
   I notice a fast way to predict how tall and wide this rectangle will be.
   I wonder how much it would cost to build a staircase if each step cost $25.
   I wonder what happens if I use 4 blocks to make the first step?
   I wonder if I can make a picture for 1 + 3 + 5 + 7, etc.

3. **After solving the problem**
   I notice that grouping numbers and changing their order can make adding a lot easier.
   I notice that sometimes you can turn an adding problem into a multiplying problem, even when the numbers are not all the same.
   I wonder if there are *always* fast ways to add numbers that go up in a pattern.
   I wonder if I could make my own pictures for long strings of numbers with different patterns.
   I wonder if it would be possible to make stairs out of triangles.
   I wonder if it would change the number of blocks I need if I did this.

The process of asking and answering new questions can go on for a long time.
You never stop noticing and wondering!

There is a second "Opener" on the final page of this project for students who are curious. It looks very different, but it is connected to their work on the steps. There are a few notes about it in the Sample Solutions and Math Notes, but most of it is left up to you!

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Step Up!
Directions

Explain your thinking for every question that you answer. Draw pictures when it helps.

Part 1

1. Show some different ways that you can see 1 + 2 + 3 + 4 in the picture.
2. How many blocks are in a 10-step staircase?
3. How many blocks are in larger staircases? (Try 100 or 1000 steps, for example.)

Part 2

4. How many steps can you make with 500 (or some other number of) blocks?

Part 3

5. Figure out a fast way to add strings of odd numbers such as

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19.$$

6. Look at the second “Opener” page. Notice and wonder as much as you can. Do some investigating to answer your own questions.

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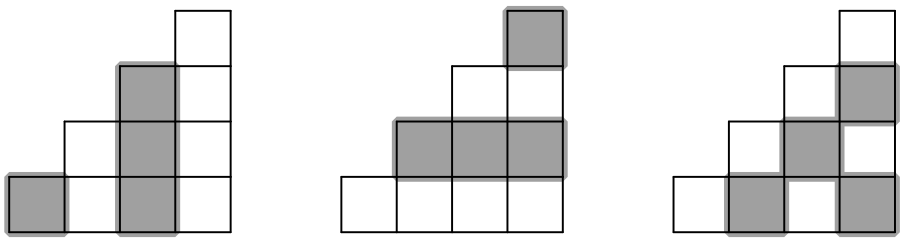
Step Up!
Math Notes and Solutions

Different ways to see 1 + 2 + 3 + 4



1 + 2 + 3 + 4 1 + 2 + 3 + 4 1 + 2 + 3 + 4

Building larger staircases

It takes 55 blocks to make 10 steps.
It takes 5050 blocks to make 100 steps.
It takes 500,500 blocks to make 1000 steps.

(Can you see patterns that will help you predict the number of blocks you would need for ten thousand or a hundred thousand steps?)

Examples of strategies

Strategy 1

Draw the staircase, and count the blocks. (This is not very practical for large staircases!)

Strategy 2

Add the numbers in order.

1 + 2 = 3
3 + 3 = 6
6 + 4 = 10

This will also take a very long time if there are a lot of steps—and you are likely to make mistakes in your calculations!

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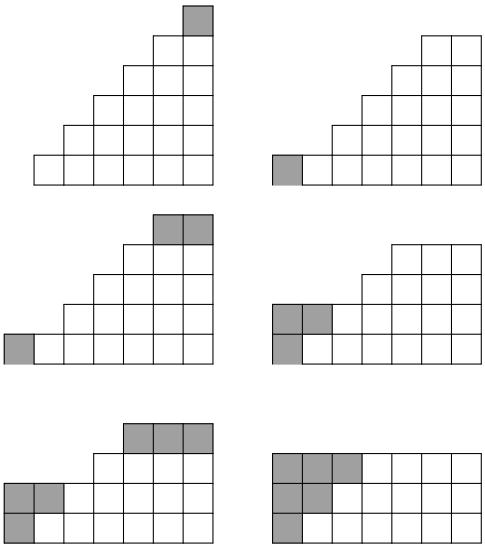
Strategy 3

Turn the staircase into a rectangle by moving blocks. This example shows a 6-step staircase. Keep moving the blocks from the top layer to the next layer on the bottom. (Read the example from left to right, top to bottom.)



Notice how $1 + 2 + 3 + 4 + 5 + 6$ turns into $(1 + 6) + (2 + 5) + (3 + 4)$, or

$$7 + 7 + 7 = 7 \times 3 = 21.$$

If you do this with 10 steps, each layer will have 11 blocks, and there will be 5 layers.

$$11 \times 5 = 55$$

If you do it with 100 steps, each layer will have 101 blocks, and there will be 50 layers.

$$101 \times 50 = 5050$$

If you do it with 1000 steps, each layer will have 1001 blocks, and there will be 500 layers.

$$1001 \times 500 = 500{,}500$$

One way to multiply these numbers is to think of groups. For example:

1001 groups of 500 =
1000 groups of 500 (or 500 groups of 1000) + 1 group of 500
500,000 + 500 = 500,500

The general pattern: The number of blocks in each layer is *one greater than the number of steps*, and there are *half as many layers as steps*. Multiplying these two numbers tells you the number of blocks!

Blocks = (Steps + 1) × (Steps ÷ 2)

When there is an odd number of steps, you will not be able to complete the final layer of steps. How can you adjust for this? Does it affect the formula above?

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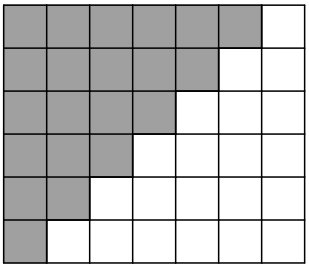
Strategy 4

Make a rectangle by making a copy of the staircase, turning it upside-down and joining it to the original staircase. Notice how it looks for a 6-step staircase (with the upside-down staircase in grey).



There are 6 layers with 7 blocks in each:

$$6 \times 7 = 42$$

Half of them belong to the original stairs:

$$42 \div 2 = 21$$

If you do this with 10 steps, you will have 10 layers of 11 blocks each.

$$10 \times 11 = 110 \qquad 110 \div 2 = 55$$

If you do this with 100 steps, you will have 100 layers of 101 blocks each.

$$100 \times 101 = 10{,}100 \qquad 10{,}100 \div 2 = 5050$$

If you do this with 1000 steps, you will have 1000 layers of 1001 blocks each.

$$1000 \times 1001 = 1{,}001{,}000 \qquad 1{,}001{,}000 \div 2 = 500{,}500$$

The general pattern: The number of blocks in each layer is *one greater than the number of steps*, and there is the *same number of layers as steps*. Multiply these two numbers to find the number of blocks in the rectangle. Take half of the result to find the number of blocks in the staircase.

Blocks = (Steps + 1) × Steps ÷ 2

Compare this formula to the one from Strategy 3.

- In Strategy 3, you divide the number of steps by 2 before you multiply.
- In Strategy 4, you multiply by the number of steps before you divide by 2.

You do the same things but in a different order, and both methods give the same answers!

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The number of steps you can make with 500 blocks

Using 500 blocks, you can make a staircase with 31 steps. (You will have 4 blocks left over.)

One idea is to imagine making blocks into a rectangle

(1) Whose length is 1 greater than its width, and
(2) That has less than (but as close as possible to) 1000 blocks.

A 31 by 32 rectangle has $31 \times 32 = 992$ blocks. You could split this rectangle into two identical staircases, each with 31 steps. Each staircase would contain $992 \div 2 = 496$ blocks.

Students will have many strategies. Most strategies are likely to involve a guess, test, and revise approach. Students will probably need to multiply two 2-digit numbers as part of their thinking process. If they have not yet learned rules for doing this, be sure to give them plenty of time and suggest that they estimate in order to reduce the number of things that they need to try. You may even allow them to use a calculator at some point—but insist that they estimate first.

Adding strings of odd numbers

Some students may do it without pictures. For example:

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 =$$
$$(1 + 19) + (3 + 17) + (5 + 15) + (7 + 13) + (9 + 11) =$$
$$20 + 20 + 20 + 20 + 20 =$$
$$20 \times 5 =$$
$$100$$

The main disadvantage to this method is that you need to know how many numbers you are adding, and it may take a while to figure this out*.

**Note*: To figure out how many numbers you are adding, you can add 1 to the last (largest) number and then take half of the result.

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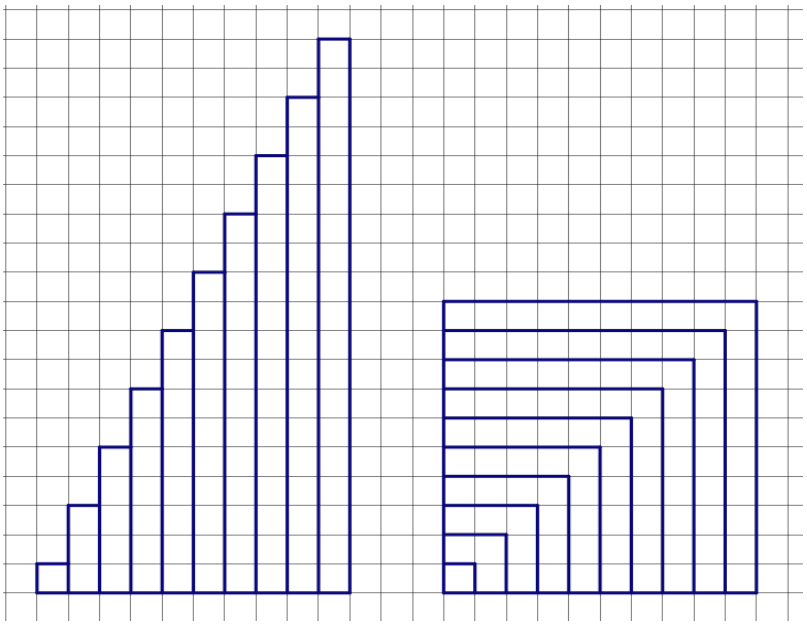
You may also use pictures. These pictures show two ways to represent

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19.$$



Each picture may lead to different strategies for finding the sum. Possible discoveries:

(1) You can find the number of blocks by multiplying the number of steps by itself.
(2) You can find the number of blocks by following the process:
- Add 1 to the final (greatest) number in the string of odd numbers.
- Divide this answer by 2. (This tells you the number of steps!)
- Multiply this answer by itself.

Notes on the second "Opener"

The second Opener (on the next page) shows a way to make every possible connection between 5 points. One point can be connected to all 4 other points (blue). The next point can be connected to 3 other points that it has not been connected to yet (red), etc.

The total number of connections is 1 + 2 + 3 + 4 = 10. This relates to the original problem of this activity! How many ways are there to connect 5, 6, 7, or more points? How can you find the answers quickly?

This second Opener may also lead to observations and questions about geometry. It may lead students to draw some very cool looking pictures and to notice many interesting things about the shapes!

Note: Be sure that students see the colors in the image. They are important!

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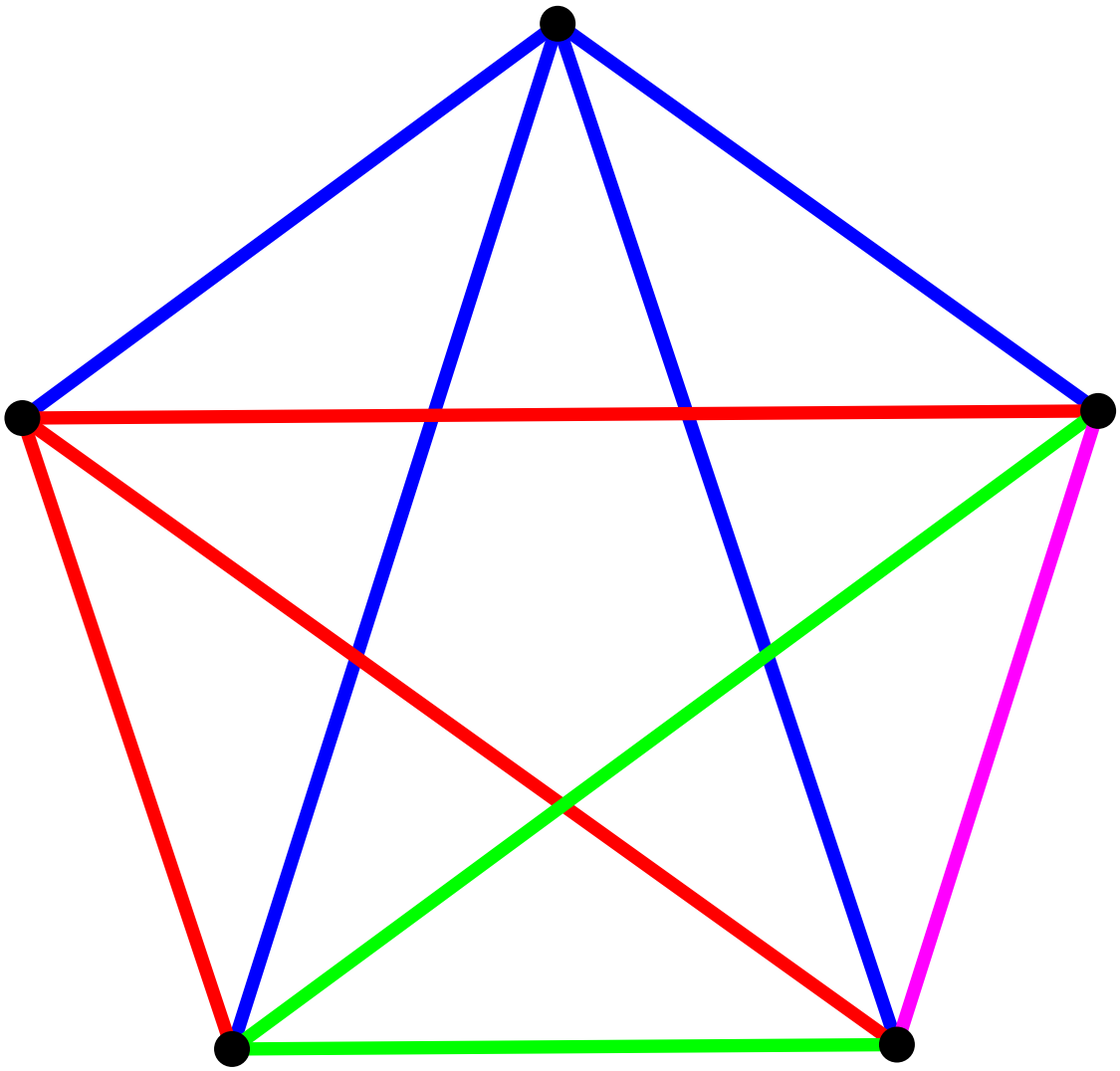
Step Up!
Another Opener

What do you notice? What do you wonder?



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